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Three-Body Model for Stripping Nuclear Reactions

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A three-body model for the deuteron stripping nuclear reactions is presented. A set of three integral equations is obtained for the wave functions of the three-body problem by introducing a decomposition into angular momentum states into the Lippmann–Schwinger equation. Simple two-particle interactions with separable potentials are used. These separable potentials reduce the three-body problem to the solution of coupled sets of one-dimensional Fredholm integral equations. The angular distributions for $^{28}Si(d,p)^{29}Si$ and $^{40}Ca(d,p)^{41}Ca$ stripping reactions are calculated. From the extracted spectroscopic factors, good agreement with the experimental measurements is obtained.

1. INTRODUCTION

Direct nuclear reactions have been shown as one of the most successful tools in studying the nuclear static properties of nuclei. The most interesting type of direct nuclear reactions are those with a particle transfer like the stripping nuclear reactions. The stripping reactions are often treated by the DWBA approximation. The most widely concerned reactions are the deuteron stripping reactions. These reactions are interesting since the deuteron is a loosely bound particle. Deuteron stripping reactions are interactions between the incident loosely bound deuteron (of a proton and a neutron) with a heavy mass target in the entrance channel leading to the exit channel of an outgoing proton interacting with a bound residual nucleus (a neutron and the target nucleus). This process is a system of two loosely bound nucleons incident on a third infinite mass target. Thus, the process of these reactions is considered as a three-body problem. The DWBA approximation with the

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perturbation approach are of great help in studying these reactions. But to understand these reactions, the three-body aspects must be taken into account. Thus, it is our aim in this work to study these deuteron stripping nuclear reactions as a three-body problem.

In the present work, a three-body model is used in studying deuteron stripping nuclear reactions. This study is based on a soluble three-body model introduced by Amado (1963) for the scattering of particles from a composite system, which is applied by Osman (1971, 1972) for studying ^{6}Li induced reactions. In the present model, the three-body problem is treated by making use of the partial wave amplitudes introduced by Glöckle and Heiss (Glöckle and Heiss, 1969; Glöckle, 1970). Then, the three-body problem of this system reduces to a Hilbert–Schmidt problem. Also, we introduce (Osman, 1973, 1974) projection operators to avoid the nonorthogonality between the different channels in these stripping reactions. In this work, we introduce simple two-particle interactions with separable potentials. The separable potentials reduce the three-body problem to the solution of coupled sets of one-dimensional Fredholm integral equations.

Here we also introduce a three-body model for studying the deuteron elastic scattering and the deuteron stripping reactions on a target nucleus X. Numerical calculations are carried out using this model. The angular distributions for the ^{28}Si$(d, p)^{29}$Si and ^{40}Ca$(d, p)^{41}$Ca stripping reactions and for the deuteron elastic scattering are calculated. Spectroscopic factors for the deuteron stripping reactions are extracted.

The different formulas of the model are introduced in Section 2. Numerical calculations and results are presented in Section 3. Section 4 is devoted to discussion and conclusions.

2. EXPRESSIONS AND FORMULAS OF THE MODEL

In the present work we study the deuteron-induced reactions. The deuteron is considered as a loose bound state of a proton and a neutron. The present process is a three-body problem of incident deuteron (proton plus neutron) on a target nucleus X. We mainly treat the following direct nuclear reactions: (i) deuteron elastic scattering

$$d + X \rightarrow d + X$$

and (ii) deuteron stripping reactions

$$d + X \rightarrow p + Y$$

in the sense that the interaction allows for the processes

$$d \rightleftharpoons p + n$$

and

$$Y \rightleftharpoons n + X$$

Let us introduce the annihilation operators for the different nuclei considered as Φ's and Ψ's. Then, the interaction Hamiltonian for the deuteron elastic scattering process (i) is given by

$$H_{\mathrm{el}} = \frac{\lambda_d}{2} \sum_{k_d, k_d'} \mathcal{V}_d \left[\left(\frac{\mathbf{k}_d - k_d'}{2} \right)^2 \right] \left\{ \Phi_{k_d + k_d'} \psi_{k_d}^{\dagger} \psi_{k_d'}^{\dagger} + \Phi_{k_d + k_d'}^{\dagger} \psi_{k_d} \psi_{k_d'} \right\} \tag{1}$$

For the deuteron stripping reaction process (ii), we have the following expression for the interaction Hamiltonian:

$$H_{\mathrm{str}} = \lambda_d \sum_{k_d, k_p, k_n} \mathcal{V}_d \left[\frac{(\mathbf{k}_n - \mathbf{k}_p)^2}{2} \right] \delta_{k_d - k_n - k_p}$$

$$\times \Phi_{k_d}^{\dagger} \psi_{k_n} \psi_{k_p} + \lambda_Y \sum_{k_n} \mathcal{V}_Y \left[k_n^2 \right] \Phi_{k_Y}^{\dagger} \psi_{k_n} \psi_{k_x} \tag{2}$$

In obtaining the expressions (1) and (2) we have used simple two-particle interactions with separable potentials (Mitra, 1962) as follows:

$$\mathcal{V}_d(q^2) = \lambda_d / (q^2 + \beta_d^2) \tag{3}$$

and

$$\mathcal{V}_Y(q^2) = \lambda_Y / (q^2 + \beta_Y^2) \tag{4}$$

where λ_d and λ_Y in equations (3) and (4) are the renormalized coupling constants.

Separable potentials $\mathcal{V}_d$ and $\mathcal{V}_Y$, as expressed by equations (3) and (4), are very useful in evaluating the deuteron elastic scattering and deuteron stripping amplitudes. With the expressions (3) and (4) for the separable potentials, and from the Hamiltonian H_{el} given by equation (1), the deuteron elastic scattering amplitude is given by

$$\langle \mathbf{k}' | T_{\mathrm{el}}(E) | \mathbf{k} \rangle = \langle \mathbf{k}' | B(E) | \mathbf{k} \rangle$$

$$+ \frac{1}{(2\pi)^3} \int d^3 k'' \frac{\langle \mathbf{k}' | B(E) | \mathbf{k}'' \rangle D_d(E - k''^2 - \varepsilon_x)}{E - k''^2 - \varepsilon_x}$$

$$\times \langle \mathbf{k}'' | T_{\mathrm{el}}(E) | \mathbf{k} \rangle \tag{5}$$

The first term on the right-hand side of equation (5) is the first Born approximation. Using separable potentials, the first Born approximation $\langle \mathbf{k}'|B(E)|\mathbf{k}\rangle$ in equation (5) is given by

$$\langle \mathbf{k}'|B(E)|\mathbf{k}\rangle = \frac{\lambda_d^2 \lambda_Y^2}{(2\pi)^3} \int d^3q \frac{\mathcal{V}_d\left[\frac{1}{4}(\mathbf{q}-\mathbf{k}')^2\right]}{E - q^2 - (\mathbf{k}'-\mathbf{q})^2 - \varepsilon_x}$$
$$\times \frac{\mathcal{V}_d\left[\frac{1}{4}(\mathbf{q}-\mathbf{k})^2\right]\mathcal{V}_Y\left[(\mathbf{k}'-\mathbf{q})^2\right]\mathcal{V}_Y\left[(\mathbf{k}-\mathbf{q})^2\right]}{E - q^2 - (\mathbf{k}-\mathbf{q})^2 - \varepsilon_x}$$
$$\times \frac{D_Y(E - q^2 - \varepsilon_Y)}{\left[E - q^2 - \varepsilon_Y\right]} \tag{6}$$

In equations (5) and (6), ε_X is the energy of the target nucleus X, ε_d is the deuteron binding energy (a proton and a neutron). D_d is the sum of the bubbles for the deuteron nucleus represented graphically in Figure 1 and is given by

$$\left[D_d(\zeta)\right]^{-1} = 1 - \frac{\lambda_d^2 \zeta}{(2\pi)^3} \int d^3q \frac{\mathcal{V}_d^2(q^2)}{(2q^2+\varepsilon_d)^2(\zeta - 2q^2 - \varepsilon_d)} \tag{7}$$

Also, ε_Y is the binding energy of the bound state of the nucleus Y (the nucleus X plus the captured neutron) and D_Y is the sum of the bubbles of

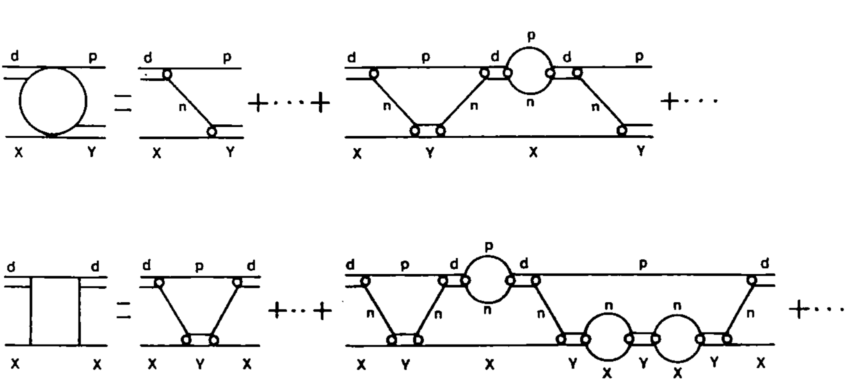

Fig. 1. The sum of graphs for deuteron stripping and elastic scattering, respectively. The circle represents the full stripping amplitude and the box the full $d - X$ amplitude. The number of bubbles and ladder rungs may be arbitrary.

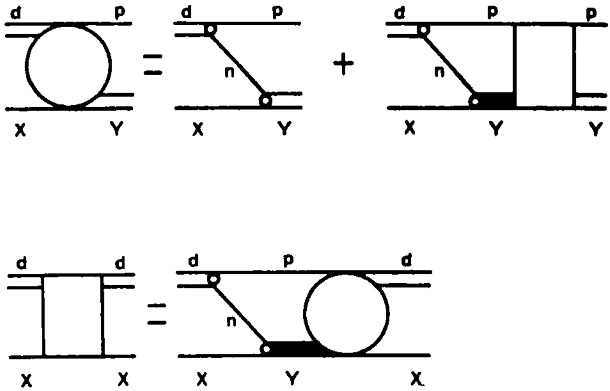

Fig. 2. The coupled integral equations for deuteron stripping and elastic scattering amplitudes, respectively.

the nucleus Y represented graphically in Figure 1 and is given by

$$[D_Y(\zeta)]^{-1} = 1 - \frac{\lambda_Y^2 \zeta}{(2\pi)^3} \int d^3q \frac{\mathcal{V}_Y^2(q^2)}{(2q^2 + \varepsilon_Y)^2(\zeta - 2q^2 - \varepsilon_Y)} \tag{8}$$

In fact, equation (5) is a coupled integral for the deuteron elastic scattering amplitude $T_{el}(E)$, which is very similar to the Lippmann–Schwinger (1950) equation with $B' = D^{1/2}BD^{1/2}$, where B' plays the role of the exact optical potential.

The deuteron stripping amplitude $T_{str}(E)$ can be computed completely as an integral over the deuteron elastic scattering amplitude $T_{el}(E)$. This integration is performed using the equation given implicitly by the first graph in Figure 2. Also, it is helpful to use the partial wave amplitudes in calculating the interaction amplitudes.

3. NUMERICAL CALCULATIONS AND RESULTS

In the present work, we consider the deuteron induced reactions. We consider the reactions induced by deuterons on ^{28}Si target nucleus at incident deuteron energy of 18.0 MeV and on ^{40}Ca target nucleus at incident deuteron energy of 7.0 MeV. We have calculated the angular distributions of the deuteron stripping reactions ^{28}Si$(d, p)^{29}$Si and ^{40}Ca$(d, p)^{41}$Ca. Also, we have calculated the deuteron elastic scattering on ^{28}Si as well as on ^{40}Ca nuclei. In the numerical calculations, the numerical values of the different parameters present in the two-particle interaction with separable potential presented by expression (3), are determined by varying the parameters λ_d and β_d independently to fit the deuteron binding energy ε_d taken as 2.225 MeV. These parameters are related by the expression

$$m\lambda_d = 8\pi\beta_d\left(\sqrt{m}\,\varepsilon_d + \beta_d\right)^2 \tag{9}$$

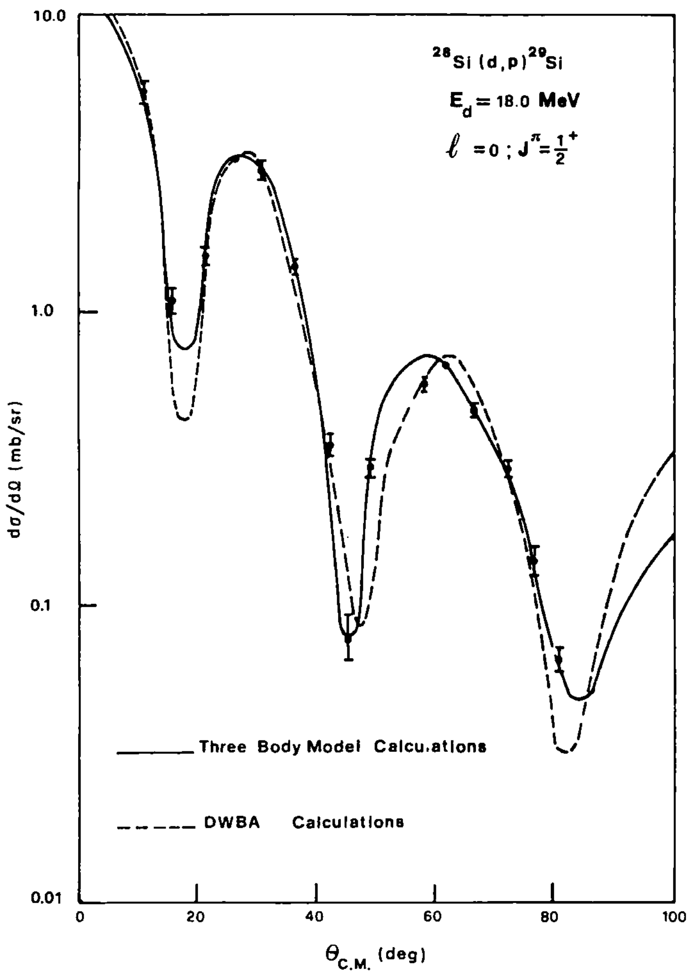

Fig. 3. The angular distributions of the nuclear stripping reaction $^{28}Si(d, p)^{29}Si$ of incident deuteron energy 18.0 MeV, leaving the residual nucleus ^{29}Si in its ground state. The solid curve is our present three-body calculations. The dashed curve is the DWBA calculations. The experimental data are taken from Mermaz et al. (1971).

Also, with a similar expression, we get the numerical values of the parameters λ_Y and β_Y presented in expression (4) for the different reactions considered.

Including the spin in the present cases is straightforward, since the coupled integral equations which leads to it are simple and the calculations are manageable on the computer.

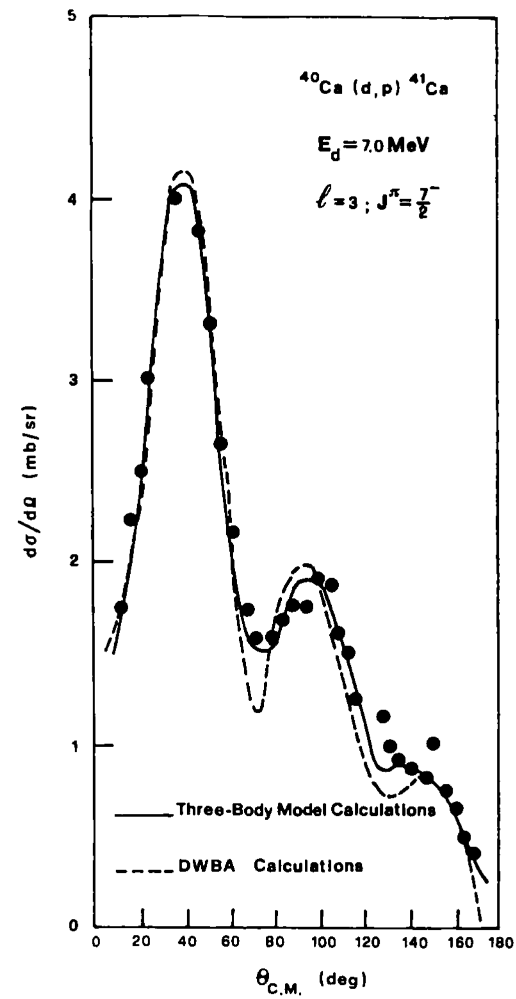

Fig. 4. The angular distributions of the nuclear stripping reaction $^{40}Ca(d, p)^{41}Ca$ of incident deuteron energy 7.0 MeV, leaving the residual nucleus ^{41}Ca in its ground state. The solid curve is our present three-body model calculations. The dashed curve is the DWBA calculations. The experimental data are taken from Lee et al. (1964).

Table I. Extracted Spectroscopic Factors

Reaction	Incident energy (MeV)	l	J^{π}	Spectroscopic factors: Present three-body model	DWBA calculations[a]	Previous work
$^{28}Si(d, p)^{29}Si$	18.0	0	$\frac{1}{2}^{+}$	0.9989	0.9841	0.53
$^{40}Ca(d, p)^{41}Ca$	7.0	3	$\frac{7}{2}^{-}$	0.9326	0.8421	0.742

[a]See Osman and Zaky, (1976).

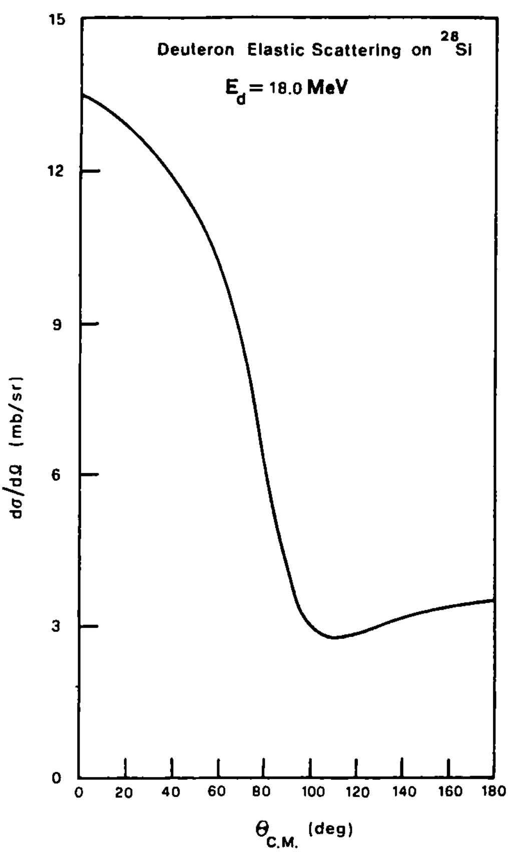

Fig. 5. The angular distribution of deuteron elastic scattering on ^{28}Si.

Numerical calculations are carried out using the present theoretical formulas. The angular distributions of the deuteron stripping reactions $^{28}Si(d, p)^{29}Si$ and $^{40}Ca(d, p)^{41}Ca$ are calculated at incident deuteron energies of 18.0 and 7.0 MeV and are introduced in Figs. 3 and 4, respectively. The experimental data for the two reactions considered are represented in Figures 3 and 4 by points and are taken from Mermaz et al. (1971) and Lee et al. (1964), respectively. The present theoretical three-body model calculations are represented by the solid curves. For the purpose of comparison, we introduce (Osman and Zaky, 1976) the perturbation approach calculations with the DWBA approximation in the same figures and represent them by dashed curves. From the present calculations, the spectroscopic factors are extracted for the different reactions considered. The values of the extracted spectroscopic factors are listed in Table I. Also, we introduce in Table I the

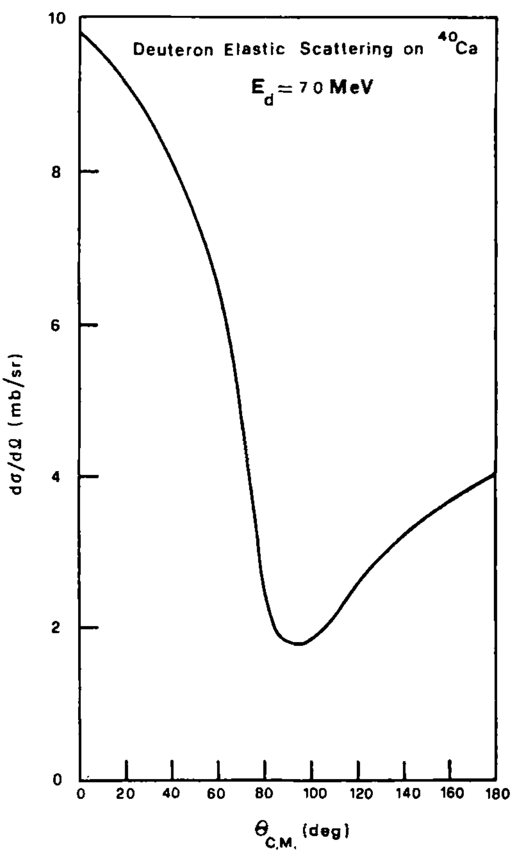

Fig. 6. The angular distribution of deuteron elastic scattering on ^{40}Ca.

spectroscopic factors of these reactions extracted from DWBA calculations (Osman and Zaky, 1976). Meanwhile, the previously obtained values of the spectroscopic factors are introduced for the purpose of comparison.

The angular distributions of the deuteron elastic scattering are numerically calculated. We consider the deuteron elastic scattering on the target nucleus ^{28}Si as well as the ^{40}Ca nucleus. The present theoretical three-body model calculations of the deuteron elastic scattering on ^{28}Si and on ^{40}Ca are introduced in Figures 5 and 6, respectively.

4. DISCUSSION AND CONCLUSIONS

In the present calculations we consider the deuteron elastic scattering and stripping reactions on a three-body model basis. The present theoretical calculations are shown in Figures 3–6. In Figures 3 and 4, the present three-body model calculations of the angular distributions for the deuteron stripping reactions are compared with the experimental data. Good agreement between our results and the experimental measurements is obtained. So, better extracted spectroscopic factors are obtained from the present three-body calculations. The present extracted spectroscopic factors are listed in Table I and compared with previously obtained values. Also, deuteron elastic scattering angular distributions have been calculated and are shown in Figures 5 and 6.

The present calculated angular distributions agree well with the experimental stripping results and give the typical stripping pattern. The angular distributions show an increase at both the forward and backward angles, with peaks at the intermediate region. The angular distributions of the deuteron elastic scattering shown in Figures 5 and 6 and also for the deuteron stripping reactions as shown in Figures 3 and 4, introduce a considerable backward peak, which is a characteristic of an exchange mechanism. Thus, the exact consequences of allowing three bodies to interact as a function of the parameters of the model give angular distributions of the sort of direct reaction mechanism, but with more structure than that given by the Born approximation.

This shows the merits of this model: First, in spite of its simplicity, it takes account explicitly of the nonadiabatic effects of the interaction. Second, since the equations derived here can be put in the form of Lippmann–Schwinger (1950) equations, or equivalent Schrödinger equations, then they are an exact optical model.

Thus, we can conclude that this model should make a good theory for deuteron stripping, but rather to make an exact theory which has the essential Born approximation of stripping.

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